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AN EXPERIMENTAL INVESTIGATION OF HEAVY VORTEX-RINGS IN HOMOGENOUS SURROUNDINGS

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In this paper two series of experiments on vortexrings moving vertically upwards have been performed.

The rings were made heavier than their surroundings by mixing sugar and dye with water.

The investigation has been done with seven different densities.

The first series of experiments indicated that the movement of the rings could be divided into three different periods.

1) Laminar period
2) Transition period
3) Turbulent period

Investigations were concentrated on the first and second periods. The maximum height of rise for a laminar ring was plotted as a function of the circulation at the formation of the ring. (Figure 5 - 11)

The maximum height of rise was defined to be the point where the ring for the first time lost dye to its surrounding. (Phot.3 and 4)

As the laminar rings moved upward, one could, if the circulation was suitable, observe growing peaks on the rear of the ring.

For a closer investigation of this process, it was necessary to do experiments with rings where only the core where marked with dye.

It then showed that the peaks were a result of standing waves on the core. These were standing waves with growing amplitude. The number of waves as well as the growth rate of the amplitude was closely connected with the circulation.

The second series of experiments was concerned with the formation of waves on the core, and the results are plotted on Figure 12. The numbers on the curve are related to the number of waves observed.

The classical theory of vortex-rings as given by Lamb [1], is built upon assumptions of such a kind that a connection between theory and experimental results hardly can be expected to exist. This of course is due to the fact that the viscosity, which is neglected in the theory, will have far reaching consequences for the movement of the ring.

However, one can get a better connection with experimental observations by using the method of V. Bjerknes and E. Høiland [2].

This method shows that even in an non-viscous fluid there can exist helical stream-lines around the core. Such stream-lines are very easily observed in real fluids and the phenomenon may contain the source of the instability causing the formation of waves on the core.

In the theory the bounding surface is supposed to be closed, but in the experiments the surface is almost always seen to be open. The effect of the stream-lines on the surface is discussed in the experimental results. (Gravity transition).

An approximate value for the circulation can be got by assuming that the vorticity produced in the boundary-layer in the mouth-piece is concerved during the formation of the ring, and that the boundary-layer forms the core of the ring.

This can only be expected to be a first approximation, because the viscosity will cause fluid with opposite vorticity to be entrained in the core.

When integrating around the boundary-layer, simple calculation shows that the circulation can be written as

$$\varkappa \cong U_o X$$

U_o is the mean velocity in the mouth-piece
X is the height of the injected fluid.

The cross section of the core will growe as a result of vorticity diffusion and a measure of the radius in core seems quite impossible.

Apparatus.

The apparatus used in these experiments consisted of a rectangular tank made of plexiglass, and connected to it, a pressure-tube.

Total volum of the tank was 144 liter $(3 \times 4 \times 12)\mathrm{dm}^3$, but it was not necessary to use the total height because the laminar rings only moved about 50 cm. However, the water height in the tank was held constant at 95 cm , to prevent the rings from being influenced by the water surface.

With the use of a mouth-peice, the pressure-tube was connected to the tank. (Fig.1) To produce an impulse use was made of a piece of iron mounted at the end of a brass rod. This hammer

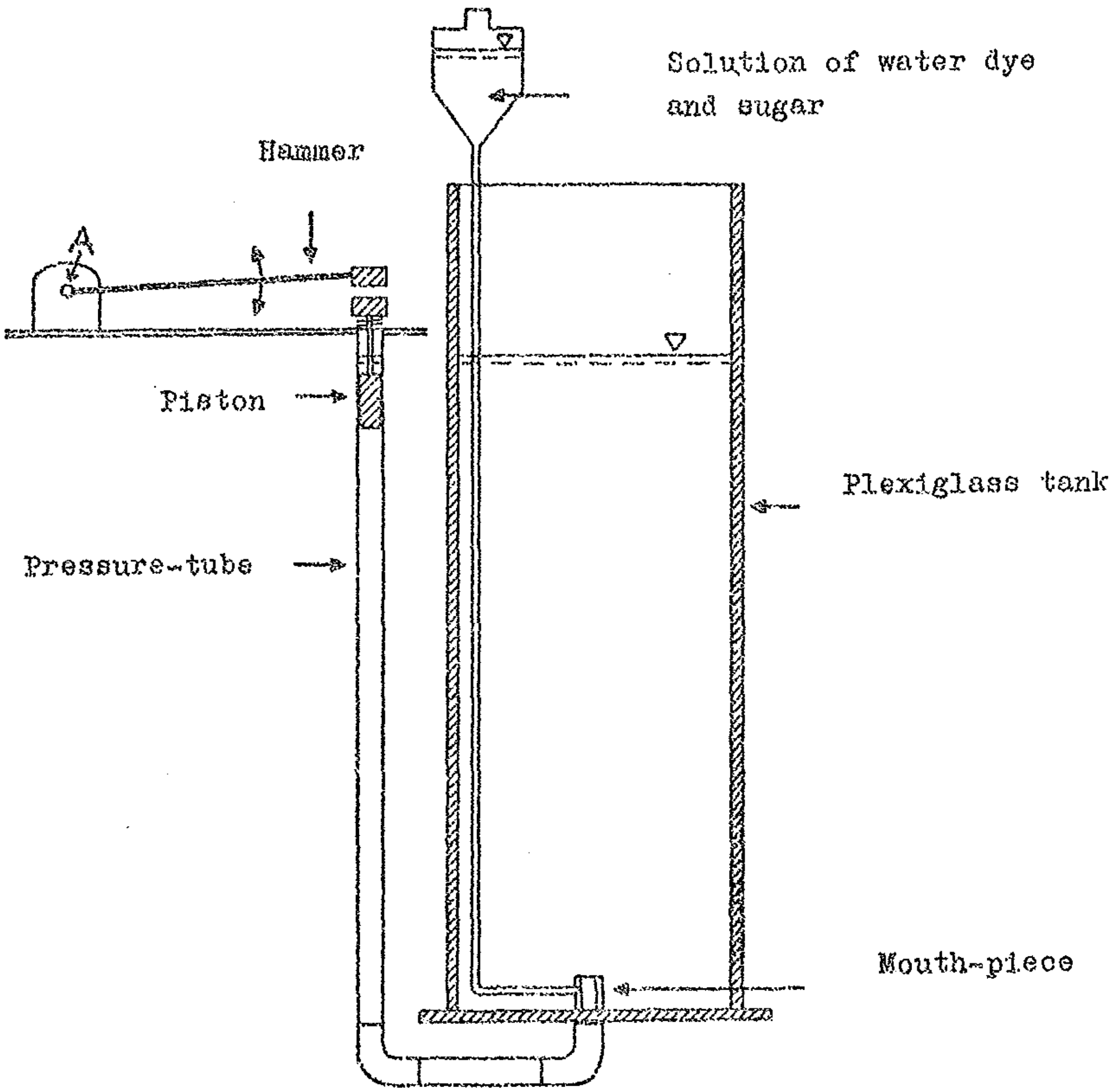


Fig. 1

could rotate about a fixed point A .

The fluid was injected in the tank by letting the hammer strike the top of a piston. At the water speed used in this investigation, the water reacts as almost completely incompressible. The impuls from the hammer will then instantaneously be transmitted to the mouth-piece.

The orifice diameter in the mouth-piece was 2 cm, and the stroke of the piston 0.4 cm. This combination gave rings with a volum of 6.75 cm^3.

By connecting a linear differential transformer to the piston and connecting this transformer to a Bruch Mark recorder, it was possible to get the speed of the piston and the mean speed of the injected water in the orifice.

It was of very great importance to inject the same amount of fluid in every ring. For this purpose the bruch recorder with a sensitivity of 20 millivolts was very well suited for adjusting the initial position of the piston.

The mixture of sugar and dye was supplied to the mouth-peice through a brass pipe on the inside of the tank. This was done to get the same temperature in the solution as in the ambient fluid. Doing this would ensure that the density difference was only due to the added sugar and dye.

The movement of the rings was criticaly dependent on the homogenity of the solution in the orifice. When the solution was incompletely distributed, one could observe a marked tendency towards an earlier breaking up of the rings. It was therefore necessary to let the solution remain in the orifice for about 5 minutes. This would cause a damping of the mechanical disturbances, and also would give time for the solution to get properly homogeneous.

It was also necessary to change the surrounding water after 3 or 4 rings, so that the solution should not disturb the new injected rings.

Between any two rings there was an interval of 10-15 minutes, ensuring that the velocityfield from earlier rings had died out.

To prevent the rings from being disturbed by pollutions in the surrounding fluid, the water had to be carefully filtrated before use. Nevertheless some of the pictures (1,3,4) seemed grainy, but this may also be a concequence of optical effects from water-plexiglass-air.

The closeup pictures are taken with the camera against the side of the tank.

By fixing a cm-scale on each side of the tank, the readings of the transition heights could be taken with an uncertainty of 1 cm.

Experimental results.

As is seen the curves can be said to have the same form, but the location of the characteristic maximum varies from the greater to the smaller density differences.

In order to get a better understanding of the phenomenon dominating the movement on the two branches of the curve, it has been very fruitful to consider the ratio of inertial-force represented by circulation, to gravity-forces represented by the density-difference between the ring and the surrounding fluid.

During the laminar part of the movement the gravity-forces remains constant, while the inertial-force continually decrease, as a result of an increase in the potential energy and entrainment of fluid. (Photos 1 and 2)

Since inertial-forces are a measure of the circulation, the circulation will continually decrease leading to a lower velocity of rise.

The lower part of the left hand branch on the curves for the heavier rings is characterized by the socalled gravity-transition.

That means transition caused by the density-difference. At these transitions the ratio of inertial - to gravity-forces is small, indicating that gravity is dominating.

Before such a transition takes place, fluid is entrained from behind. (Fig.2)

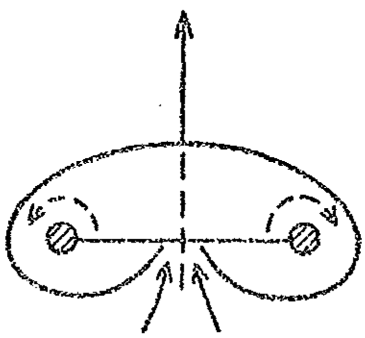


Fig.2

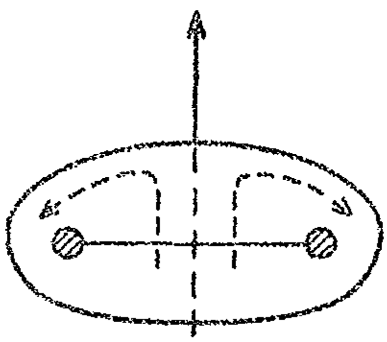


Fig.3

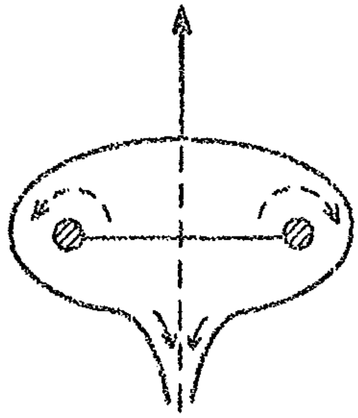


Fig.4

After a certain interval of time the velocity field from the core has been so weakened that the entrainment stops. (Fig.3) The next step is that gravity will cause a shedding of dye from the ring. (Fig.4)

In this kind of transition nothing indicated the interaction of waves from the core.

Higher up on the same branch there will be a marked influence from wave generation. This sort of transition can be said to be of the combined type. Transitions of the combined type are characterized by a greater ratio of inertia to gravity. Photos 3 and 4 show the combined transitions.

On the righthand branch of the curve transitions due to wave-generation on the core are the only existing.

For small density differences the curves start at the combined transitions. This is because at even the smallest circulation produced by the apparatus - the ratio of inertial- to gravity-forces is to great for the gravity transition to exist.

The duration of the transition period decreased with increasing circulation, and when the circulation became very fast, the transition period seemed to disappear.

After the transition period the movement of the ring could take very different forms.

Relatively small circulations could give a total disintegration in the transition period. By increasing the velocity one could get a quite new form of movement, the turbulent stage. The structure of the ring in the turbulent period was very difficult to investigate, both because of the fast circulation and the mixing of the accompanying fluid with the surrounding fluid. However, the turbulent rings seemed very stable and could move 95 cm without any other change than that caused by the entrainment of fluid.

These results refer to rings when the core as well as the accompanying fluid consisted of heavy solution.

The experiments made with rings, where only the core was marked with dye, showed how the waves could growe under the right conditions. (Photos 5-8)

There was never observed less that 4 waves, and because of the transition speed there was not observed more than 7 .

The dependence of the number of waves on the circulation is given in table 1.

Number of waves n	Circulation $\varkappa$ in cm^2/sek.
$n = 4$	$\varkappa \simeq 20 - 25$
$n = 5$	$\varkappa \simeq 25 - 29$
$n = 6$	$\varkappa \simeq 29 - 33$
$n = 7$	$\varkappa \simeq 33 - 37$

Table 1.

If $\varkappa < 20\ \mathrm{cm}^2/\mathrm{sek.}$ no waves were observed. This indicates that the disturbances introduced at the injection of fluid are under a certain limiting value. Under the given circumstances it was reasonable to expect that the critical value of the circulation would be different for different densities.

The wavelength was of the form

$$\lambda = \frac{2\pi R}{n}$$

n number of waves.

R radius of the ring.

The most interesting result of these experiments was the variation of the maximum height with the number of waves.

In the circulation interval giving $n = 4$, the transition height increased with increasing circulation. In the other intervals the transition height decreased with increasing circulation. The greatest height was reached when the circulation was about $25\ \mathrm{cm}^2/\mathrm{sek.}$ or at the limit between 4 and 5 waves.

Without having it experimentally verified, because of the dye in the accompanying fluid, it is reasonable to expect that the extremum value of the transition height is connected with a circulation laying in the interval between 4 and 5 waves.

References

[1] Sir H. Lamb, "Hydrodynamics", 1932.

[2] V. Bjerknes, E. Høiland, Norsk Matematisk Tidsskrift. Hefte 1, 1941.

Acknowledgement

It is my pleasant duty to thank Prof. E. Høiland for valuable discussion during the two years I have worked with the experiments, and also to thank Laboratory Engineer B. Schieldrop for his technical assistance.

Figure legends

Figure 5-11 shows the transition height H in cm as a function of the initial circulation $\varkappa_o$ in cm^2/sek.

Figure 5

Density difference between the ring and the surrounding fluid.

Difference in weight

$\Delta\rho = 5.5^o/oo$:

$H_{max} = 24.5cm$ $\varkappa = 57.8 cm^2/sek.$

Figure 6

$\Delta\rho = 4.5^o/oo$

$H_{max} = 25.5cm$ $\varkappa = 56.7 cm^2/sek.$

Figure 7

$\Delta\rho = 3.5^o/oo$

$H_{max} = 27cm$ $\varkappa = 55.7 cm^2/sek.$

Figure 8

$\Delta\rho = 2.5^o/oo$

$H_{max} = 28.5cm$ $\varkappa = 52.5 cm^2/sek.$

Figure 9

$\Delta\rho = 1.5^o/oo$

$H_{max} = 30cm$ $\varkappa = 37.8 cm^2/sek.$

Figure 10

$\Delta\rho = 0.5^o/oo$

$H_{max} = 31cm$ $\varkappa = 29.4 cm^2/sek.$

Figure 11

$\Delta\rho = 0.2\,^{o}/oo$

$H_{max} = 32.5cm$ $\varkappa = 26.3\,^{cm^2}/sek.$

Figure 12

Transition height for rings with dye in the core

$\Delta\rho = 2.5\,^{o}/oo$

The vertical lines divide the curve in regions of different number of waves.

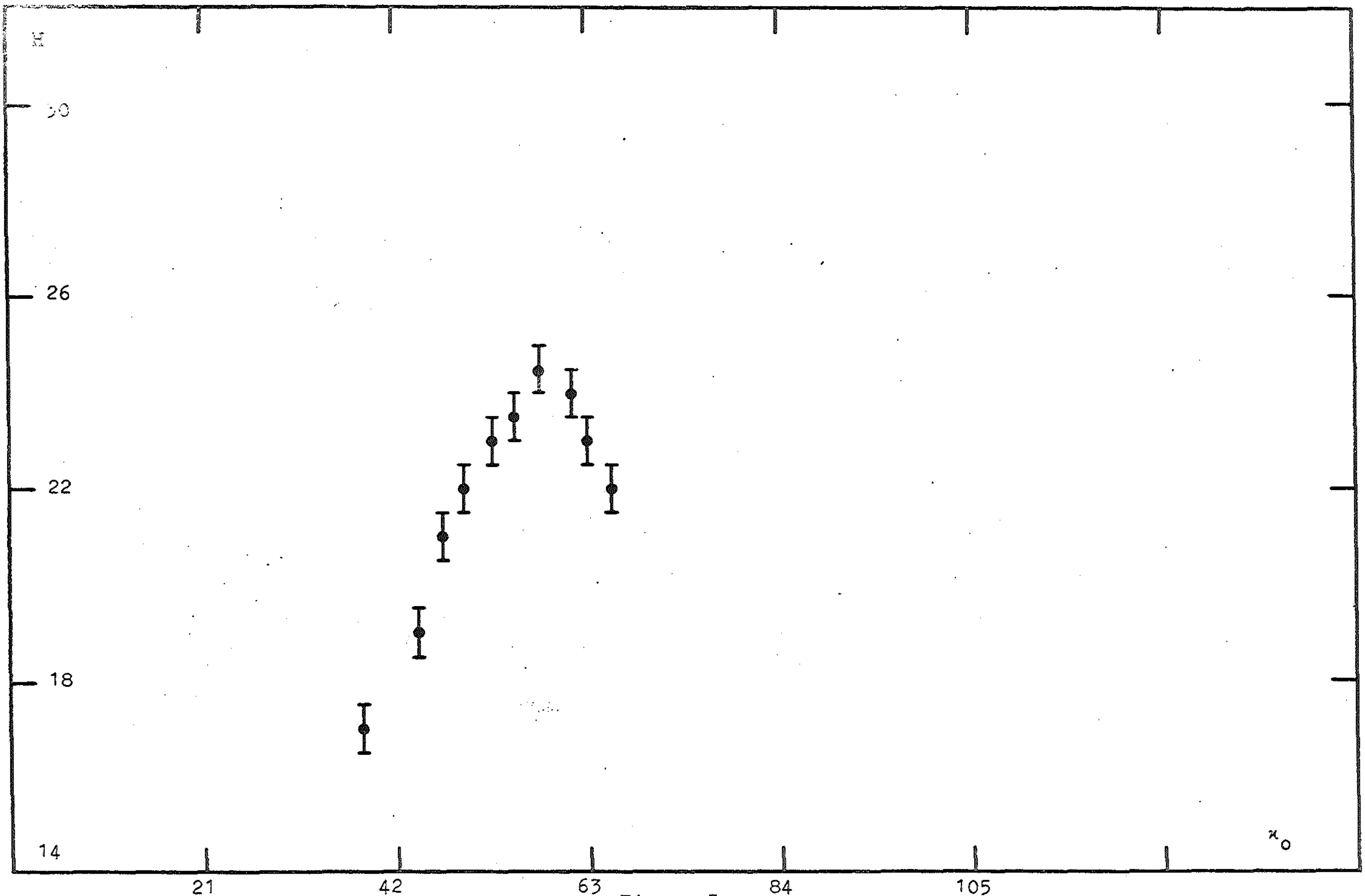


Figure 5

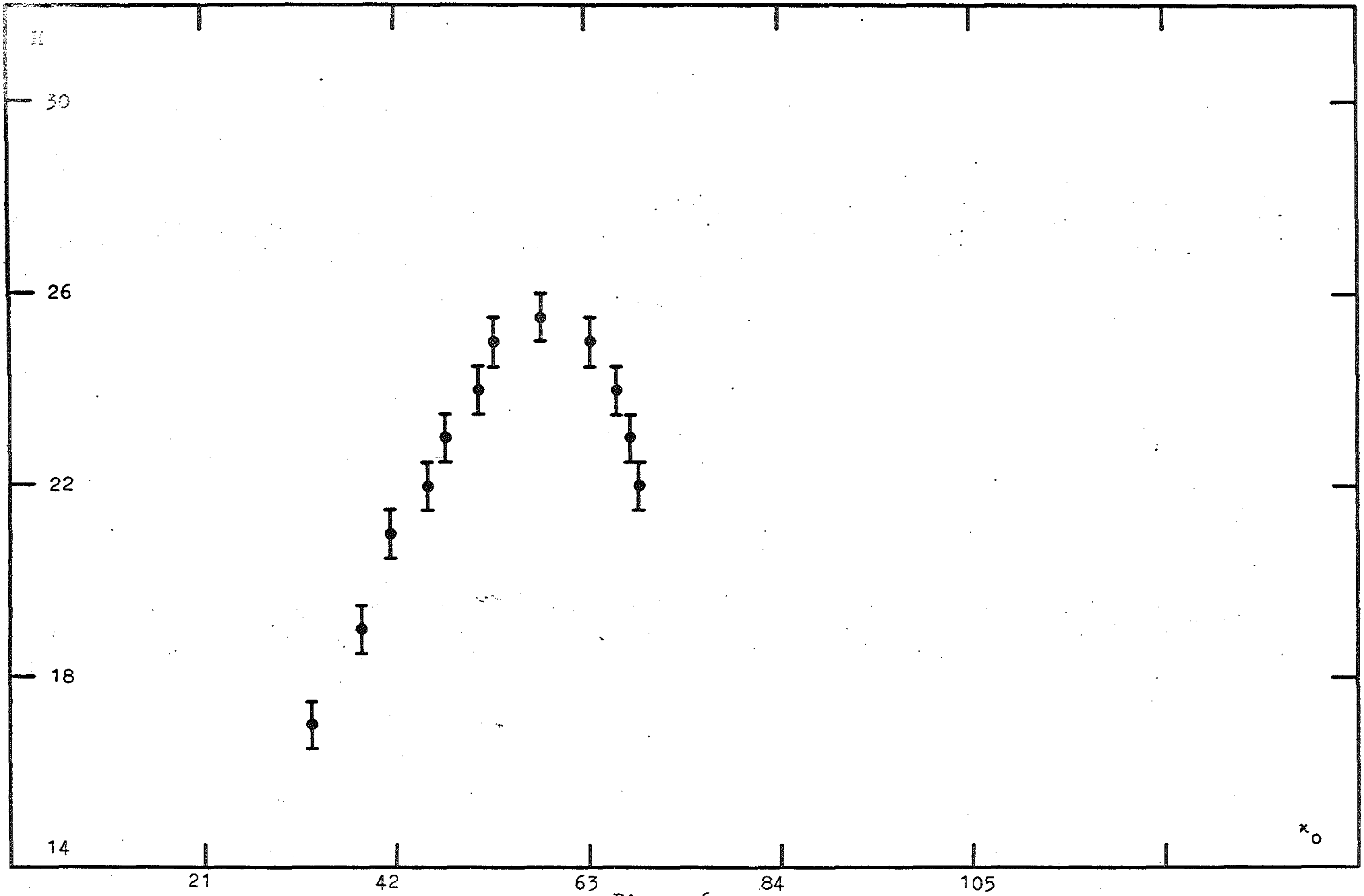


Figure 6

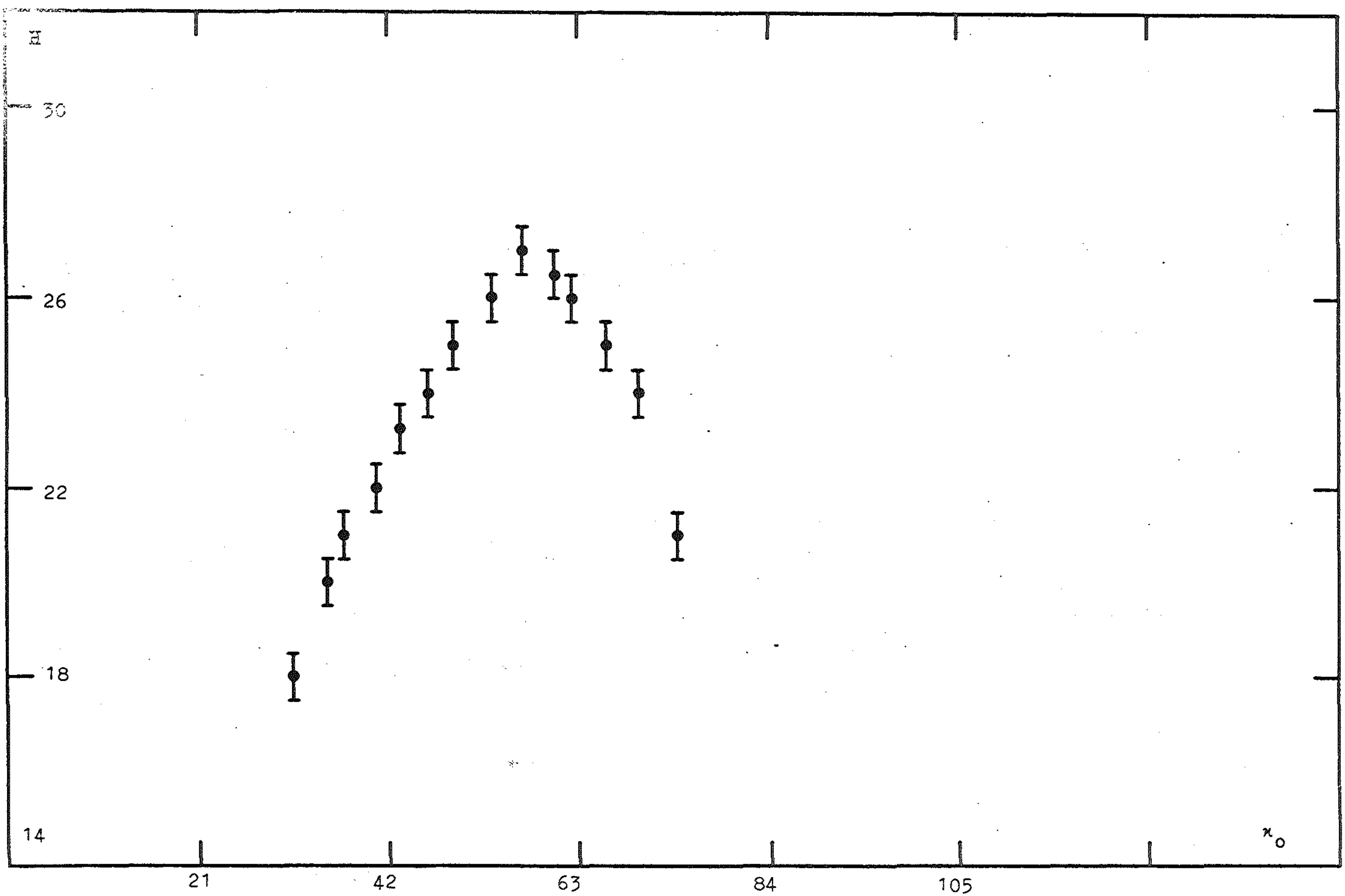


Figure 7

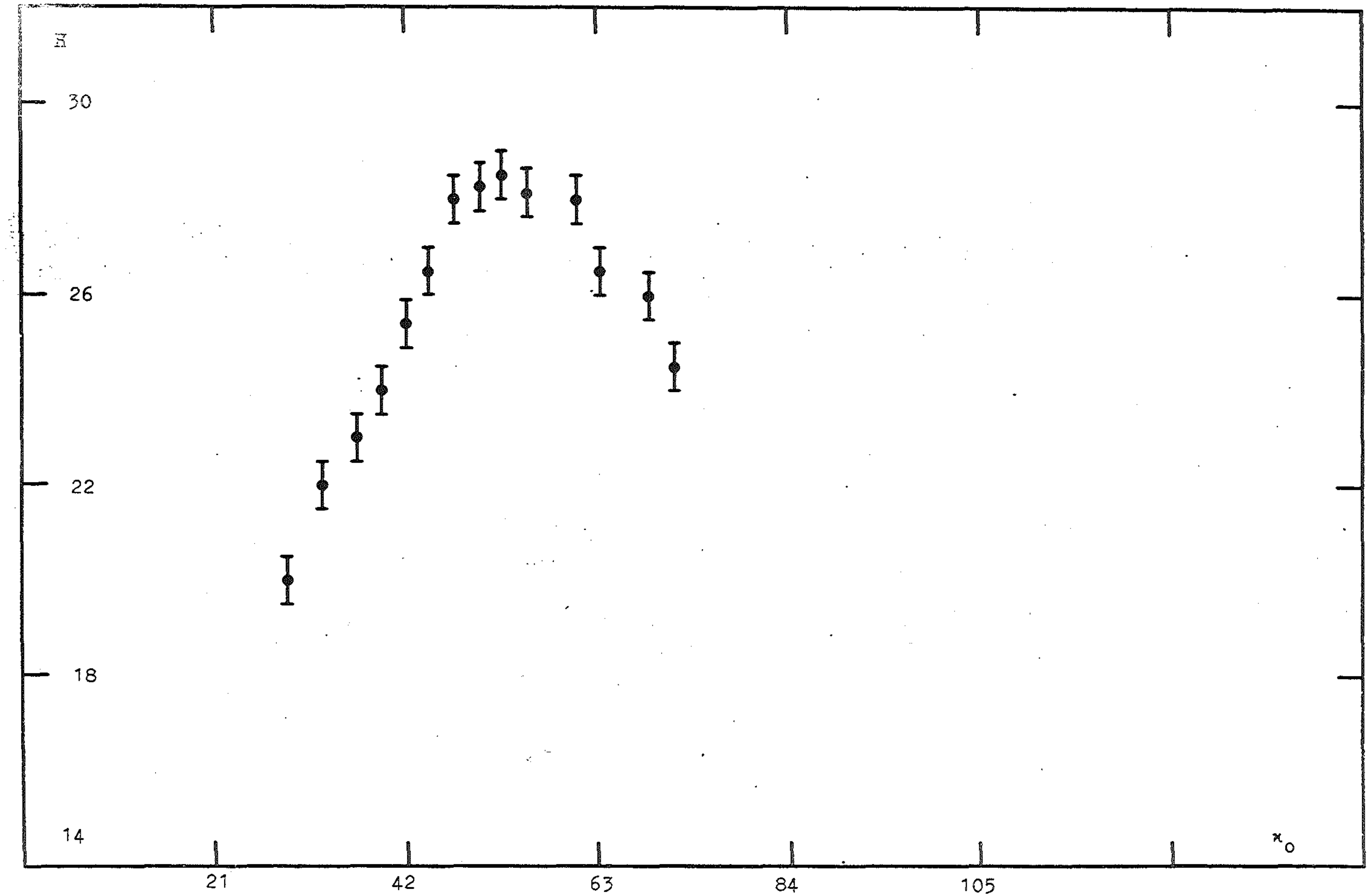


Figure 8

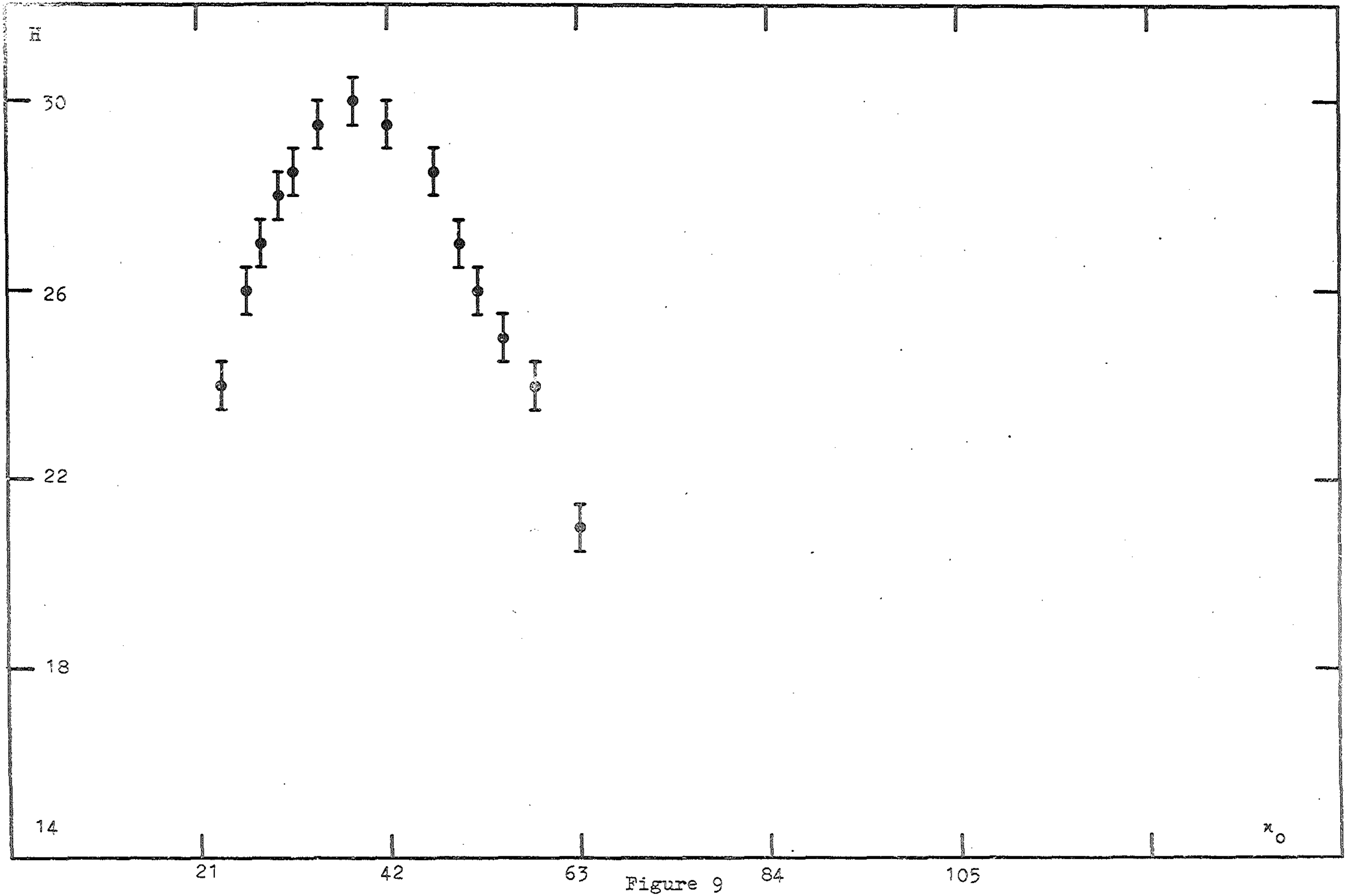


Figure 9

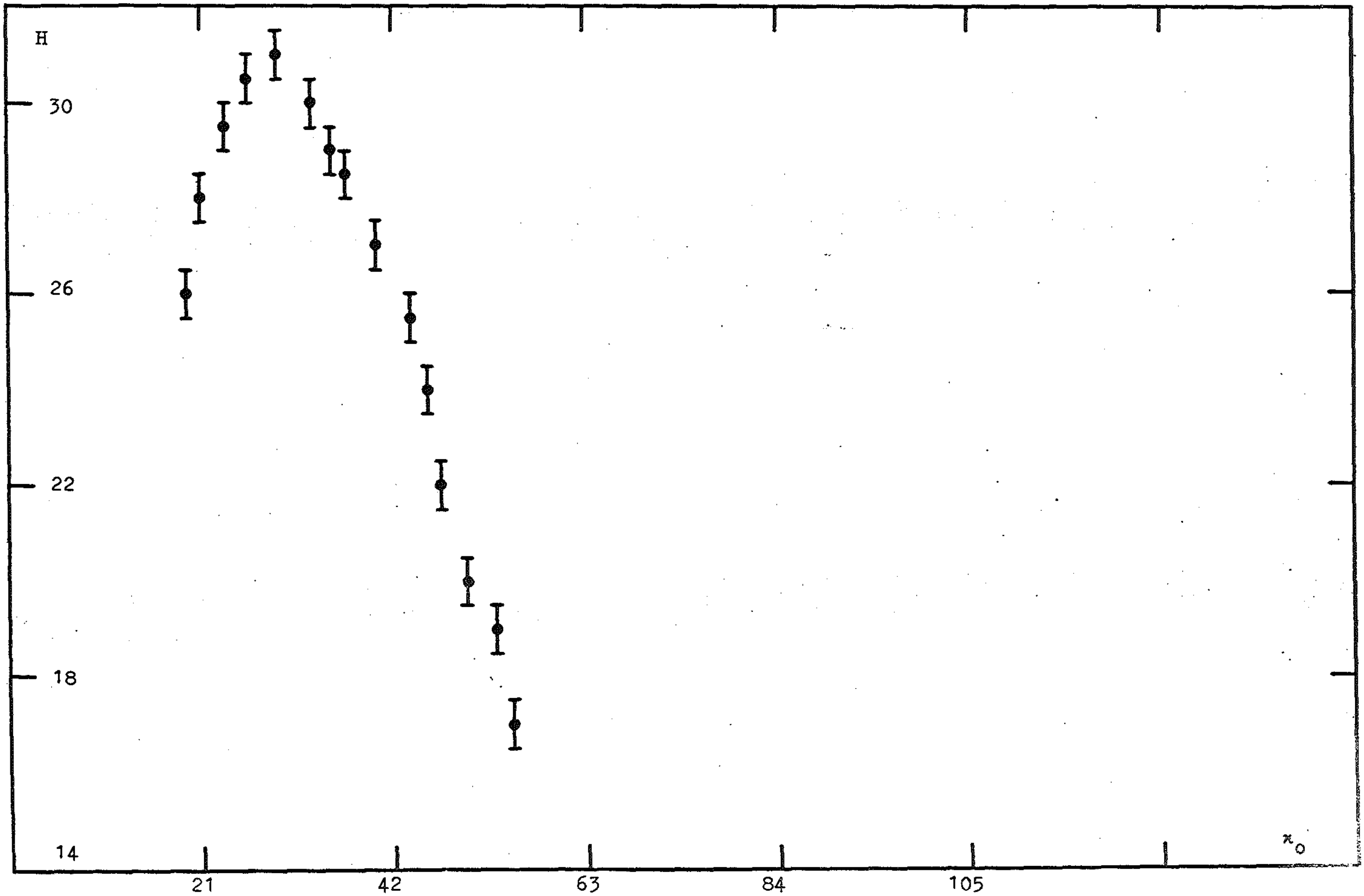


Figure 10

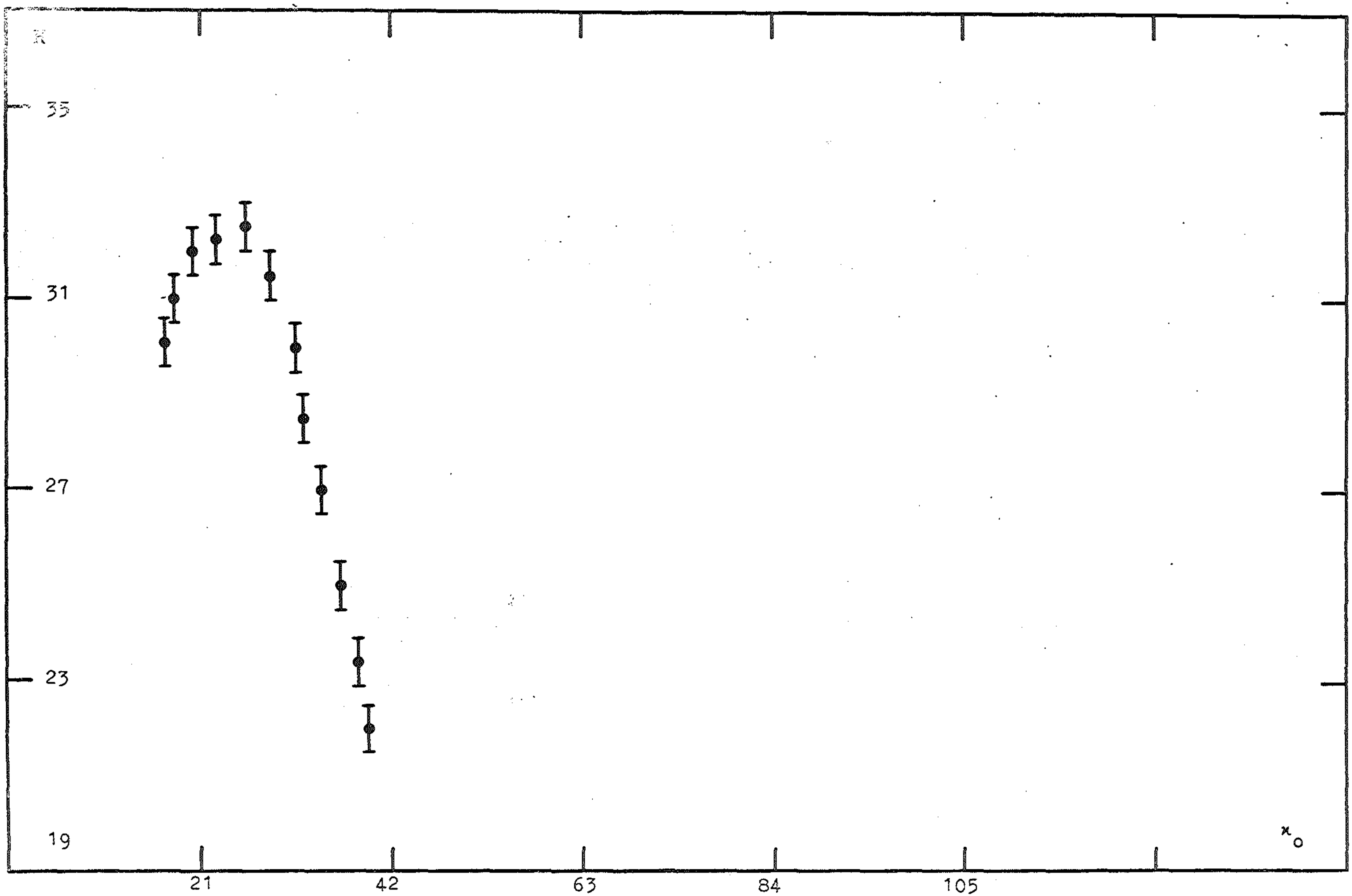


Figure 11

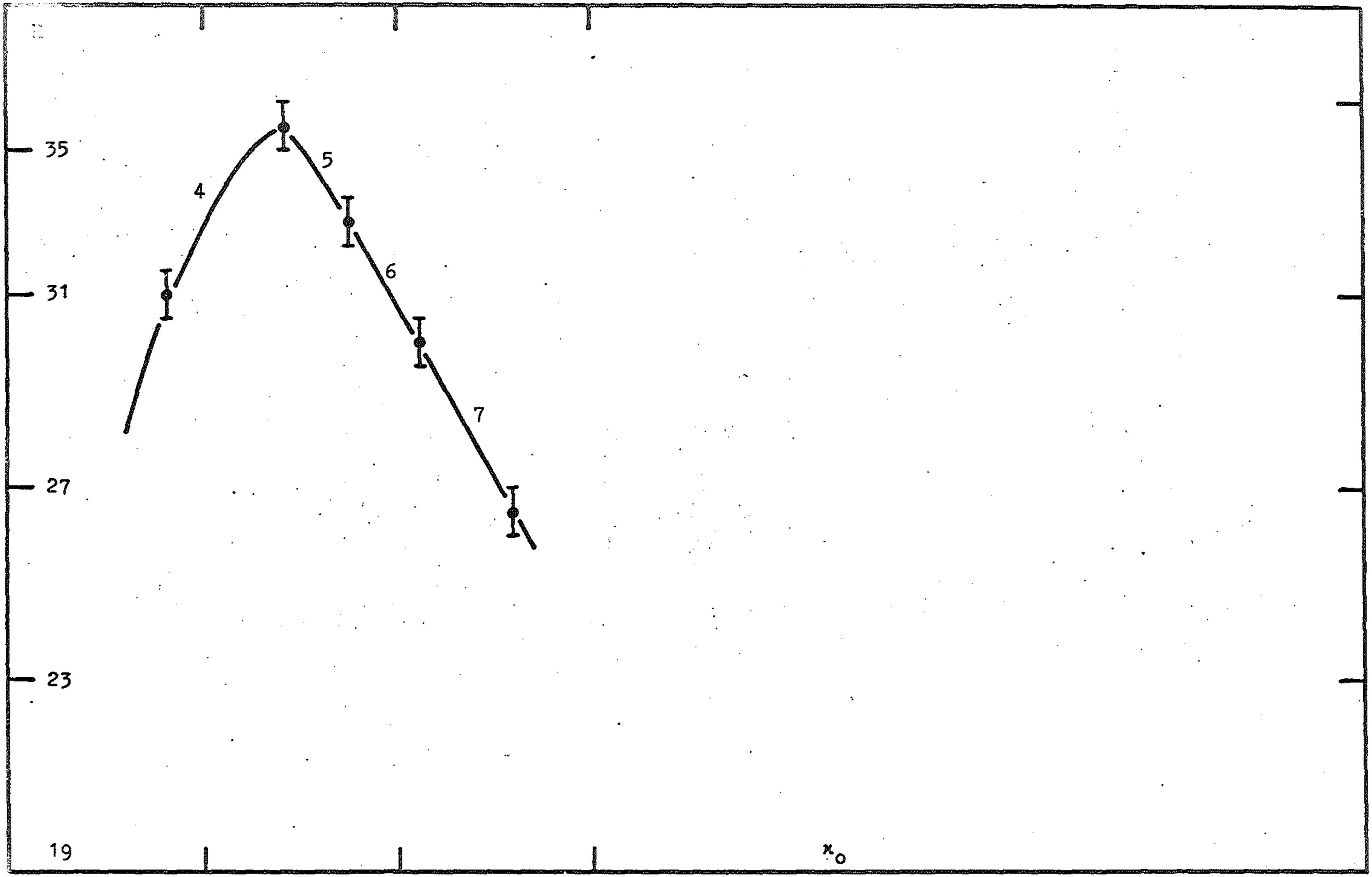
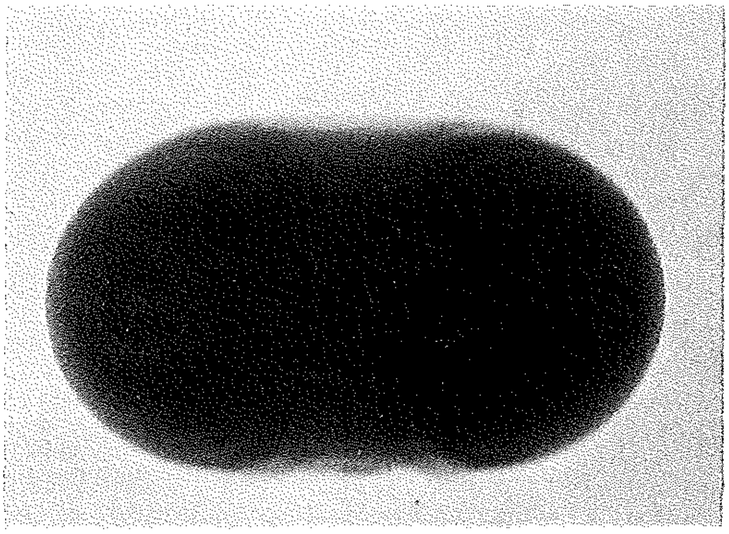
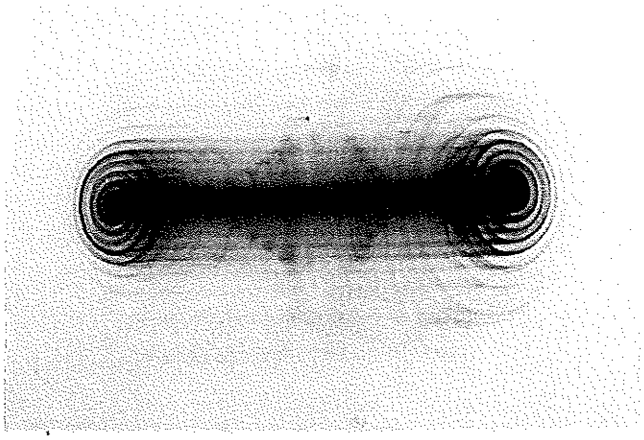


Figure 12





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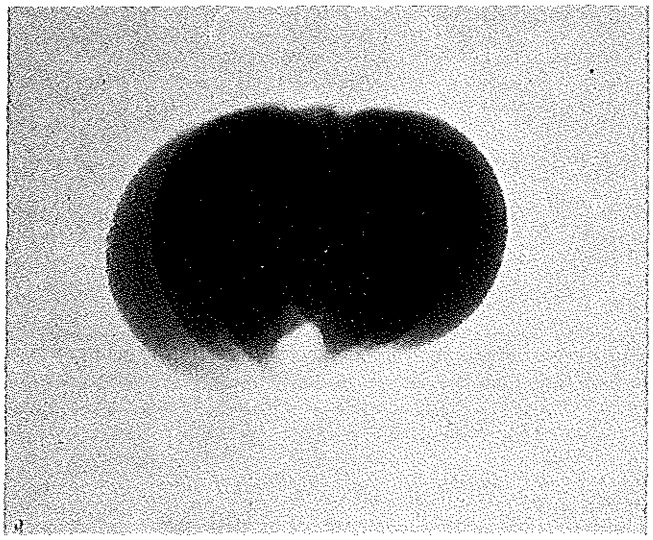
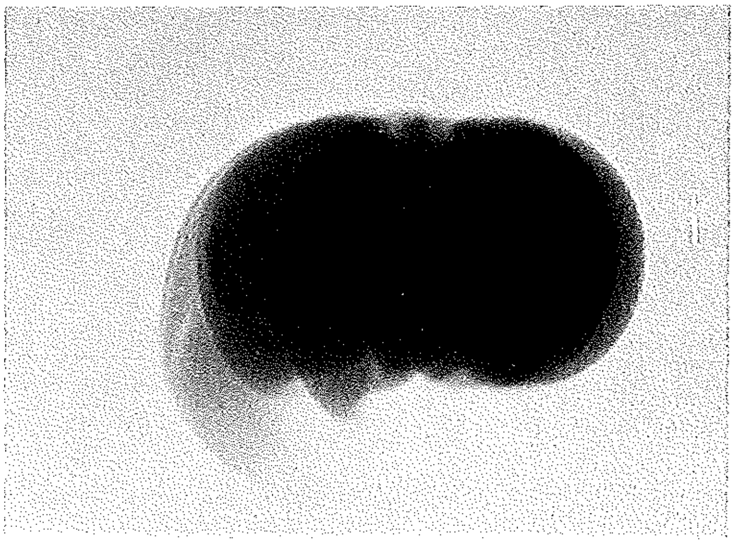
1

Laminar vortexrings with (1), and without (2), dye in the accompanying fluid.

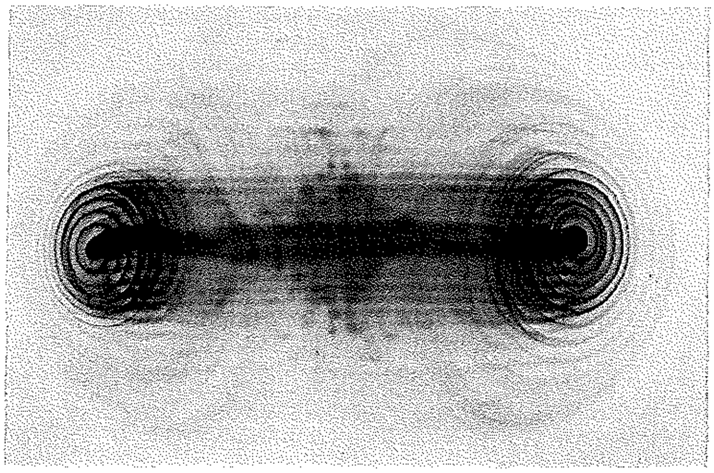




3

4

Two stages of transition where the peaks are clearly visible on the reare side of the ring.



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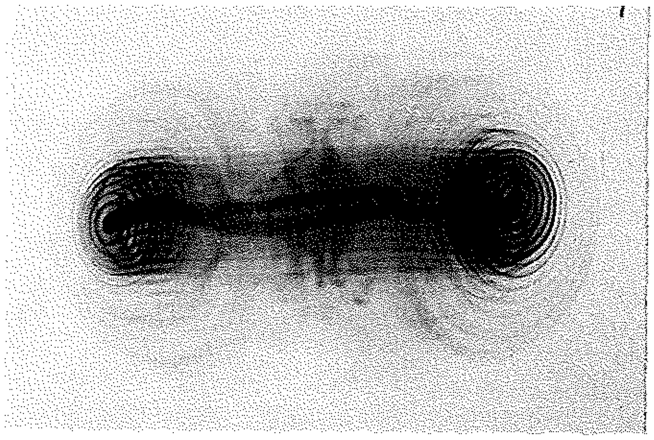

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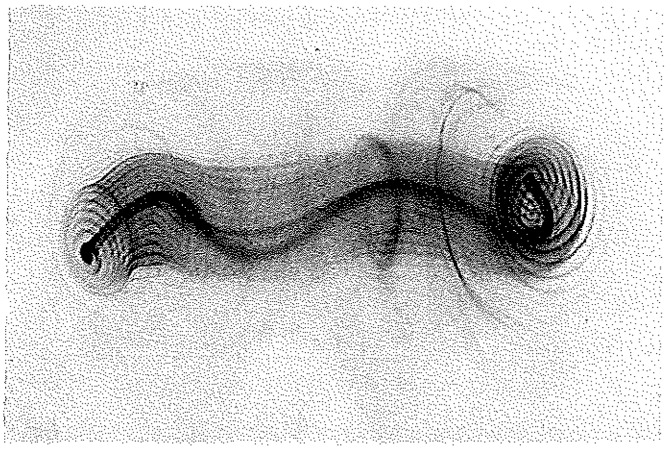

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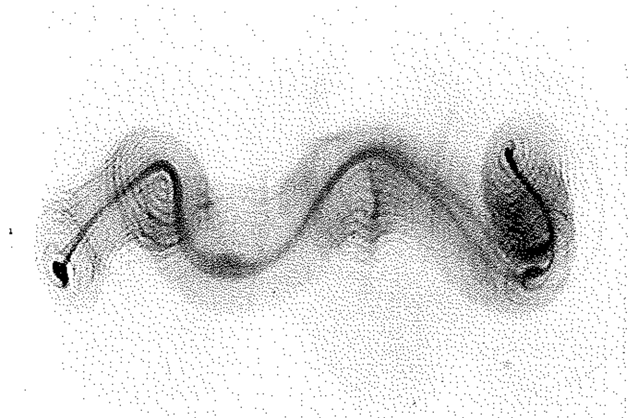

8

Finestructure of wavegeneration on the core, showing the evolution from a perfectly laminar ring (5) to a ring with fine waves (8).